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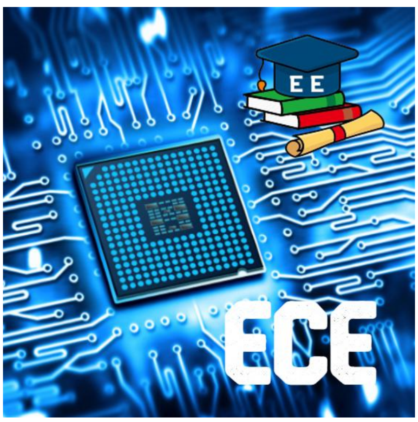

Example 13.3

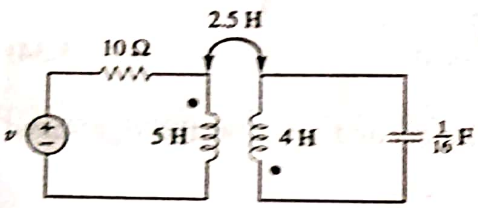

Figure 13.16
For Example 13.3.

Consider the circuit in Fig. 13.16. Determine the coupling coefficient. Calculate the energy stored in the coupled inductors at time $t = 1$ s if $v = 60\cos(4t + 30^\circ)$ V.

Solution:
The coupling coefficient is

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{2.5}{\sqrt{20}} = 0.56$$

indicating that the inductors are tightly coupled. To find the energy stored, we need to calculate the current. To find the current, we need to obtain the frequency-domain equivalent of the circuit.

$$60\cos(4t + 30^\circ) \quad \Rightarrow \quad 60\underline{/30^\circ}, \quad \omega = 4 \text{ rad/s}$$

$$5 \text{ H} \quad \Rightarrow \quad j\omega L_1 = j20\ \Omega$$

$$2.5 \text{ H} \quad \Rightarrow \quad j\omega M = j10\ \Omega$$

$$4 \text{ H} \quad \Rightarrow \quad j\omega L_2 = j16\ \Omega$$

$$\frac{1}{16} \text{ F} \quad \Rightarrow \quad \frac{1}{j\omega C} = -j4\ \Omega$$

The frequency-domain equivalent is shown in Fig. 13.17. We now apply mesh analysis. For mesh 1,

$$(10 + j20)\mathbf{I}_1 + j10\mathbf{I}_2 = 60\underline{/30^\circ} \tag{13.3.1}$$

For mesh 2,

$$j10\mathbf{I}_1 + (j16 - j4)\mathbf{I}_2 = 0$$

or

$$\mathbf{I}_1 = -1.2\mathbf{I}_2 \tag{13.3.2}$$

Substituting this into Eq. (13.3.1) yields

$$I_2(-12 - j14) = 60\angle 30^\circ \quad \Rightarrow \quad I_2 = 3.254\angle 160.6^\circ \text{ A}$$

and

$$I_1 = -1.2I_2 = 3.905\angle -19.4^\circ \text{ A}$$

In the time-domain,

$$i_1 = 3.905\cos(4t - 19.4^\circ), \qquad i_2 = 3.254\cos(4t + 160.6^\circ)$$

At time $t = 1$ s, $4t = 4$ rad $= 229.2^\circ$, and

$$i_1 = 3.905\cos(229.2^\circ - 19.4^\circ) = -3.389 \text{ A}$$
$$i_2 = 3.254\cos(229.2^\circ + 160.6^\circ) = 2.824 \text{ A}$$

The total energy stored in the coupled inductors is

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + Mi_1 i_2$$
$$= \frac{1}{2}(5)(-3.389)^2 + \frac{1}{2}(4)(2.824)^2 + 2.5(-3.389)(2.824) = 20.73 \text{ J}$$

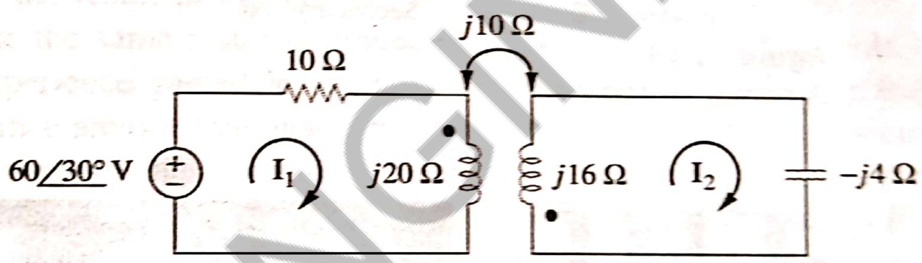

Figure 13.17
Frequency-domain equivalent of the circuit in Fig. 13.16.

Practice Problem 13.3

For the circuit in Fig. 13.18, determine the coupling coefficient and the energy stored in the coupled inductors at $t = 1.5$ s.

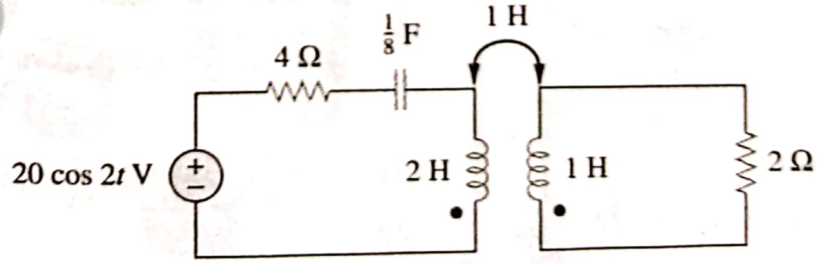

Figure 13.18
For Practice Prob. 13.3.

Answer: 0.7071, 9.85 J.

Example 13.7

An ideal transformer is rated at 2400/120 V, 9.6 kVA, and has 50 turns on the secondary side. Calculate (a) the turns ratio, (b) the number of turns on the primary side, and (c) the current ratings for the primary and secondary windings.

Solution:

(a) This is a step-down transformer, since $V_1 = 2{,}400 \text{ V} > V_2 = 120 \text{ V}$.

$$n = \frac{V_2}{V_1} = \frac{120}{2{,}400} = 0.05$$

(b)

$$n = \frac{N_2}{N_1} \quad \Rightarrow \quad 0.05 = \frac{50}{N_1}$$

or

$$N_1 = \frac{50}{0.05} = 1{,}000 \text{ turns}$$

(c) $S = V_1 I_1 = V_2 I_2 = 9.6$ kVA. Hence,

$$I_1 = \frac{9{,}600}{V_1} = \frac{9{,}600}{2{,}400} = 4 \text{ A}$$

$$I_2 = \frac{9{,}600}{V_2} = \frac{9{,}600}{120} = 80 \text{ A} \quad \text{or} \quad I_2 = \frac{I_1}{n} = \frac{4}{0.05} = 80 \text{ A}$$

Practice Problem 13.7

The primary current to an ideal transformer rated at 3300/110 V is 3 A. Calculate: (a) the turns ratio, (b) the kVA rating, (c) the secondary current.

Answer: (a) 1/30, (b) 9.9 kVA, (c) 90 A.

Example 13.8

For the ideal transformer circuit of Fig. 13.37, find: (a) the source current $\mathbf{I}_1$, (b) the output voltage $\mathbf{V}_o$, and (c) the complex power supplied by the source.

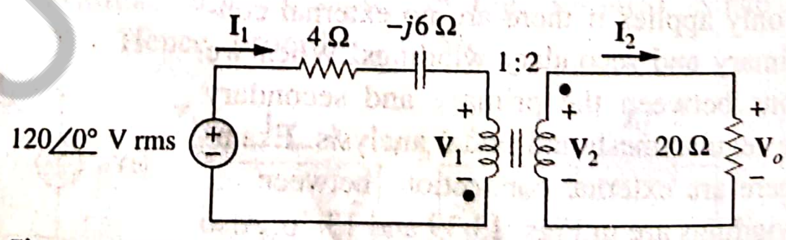

Figure 13.37
For Example 13.8.

Solution:

(a) The 20-Ω impedance can be reflected to the primary side and we get

$$\mathbf{Z}_R = \frac{20}{n^2} = \frac{20}{4} = 5 \ \Omega$$

Thus,

$$\mathbf{Z}_{in} = 4 - j6 + \mathbf{Z}_R = 9 - j6 = 10.82\underline{/-33.69^\circ}\ \Omega$$

$$\mathbf{I}_1 = \frac{120\underline{/0^\circ}}{\mathbf{Z}_{in}} = \frac{120\underline{/0^\circ}}{10.82\underline{/-33.69^\circ}} = 11.09\underline{/33.69^\circ}\ \text{A}$$

(b) Since both $\mathbf{I}_1$ and $\mathbf{I}_2$ leave the dotted terminals,

$$\mathbf{I}_2 = -\frac{1}{n}\mathbf{I}_1 = -5.545\underline{/33.69^\circ}\ \text{A}$$

$$\mathbf{V}_o = 20\mathbf{I}_2 = 110.9\underline{/213.69^\circ}\ \text{V}$$

(c) The complex power supplied is

$$\mathbf{S} = \mathbf{V}_s\mathbf{I}_1^* = (120\underline{/0^\circ})(11.09\underline{/-33.69^\circ}) = 1{,}330.8\underline{/-33.69^\circ}\ \text{VA}$$

Practice Problem 13.8

In the ideal transformer circuit of Fig. 13.38, find $\mathbf{V}_o$ and the complex power supplied by the source.

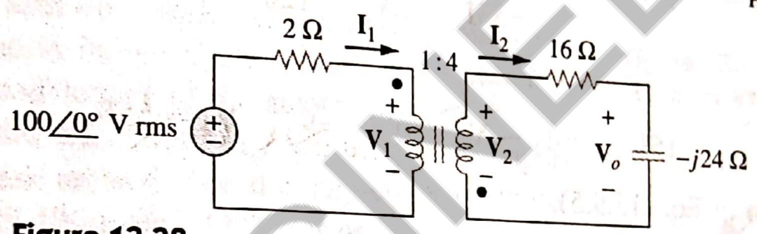

Figure 13.38
For Practice Prob. 13.8.

Answer: $178.9\underline{/116.56^\circ}$ V, $2{,}981.5\underline{/-26.56^\circ}$ VA.

Example 13.9

Calculate the power supplied to the 10-Ω resistor in the ideal transformer circuit of Fig. 13.39.

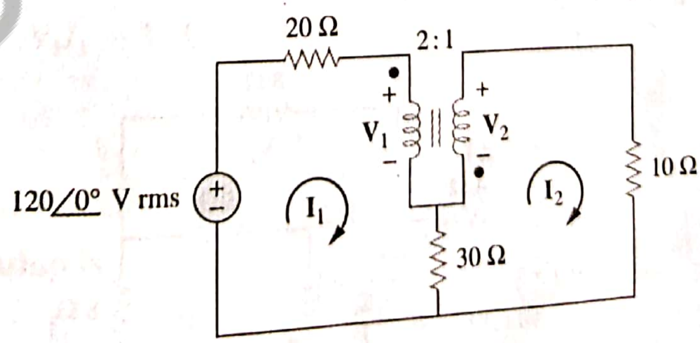

Figure 13.39
For Example 13.9.

Solution:
Reflection to the secondary or primary side cannot be done with this circuit: there is direct connection between the primary and secondary sides due to the 30-Ω resistor. We apply mesh analysis. For mesh 1,

$$-120 + (20 + 30)\mathbf{I}_1 - 30\mathbf{I}_2 + \mathbf{V}_1 = 0$$

or

$$50\mathbf{I}_1 - 30\mathbf{I}_2 + \mathbf{V}_1 = 120 \quad (13.9.1)$$

For mesh 2,

$$-\mathbf{V}_2 + (10 + 30)\mathbf{I}_2 - 30\mathbf{I}_1 = 0$$

or

$$-30\mathbf{I}_1 + 40\mathbf{I}_2 - \mathbf{V}_2 = 0 \quad (13.9.2)$$

At the transformer terminals,

$$\mathbf{V}_2 = -\frac{1}{2}\mathbf{V}_1 \quad (13.9.3)$$

$$\mathbf{I}_2 = -2\mathbf{I}_1 \quad (13.9.4)$$

(Note that $n = 1/2$.) We now have four equations and four unknowns, but our goal is to get $\mathbf{I}_2$. So we substitute for $\mathbf{V}_1$ and $\mathbf{I}_1$ in terms of $\mathbf{V}_2$ and $\mathbf{I}_2$ in Eqs. (13.9.1) and (13.9.2). Equation (13.9.1) becomes

$$-55\mathbf{I}_2 - 2\mathbf{V}_2 = 120 \quad (13.9.5)$$

and Eq. (13.9.2) becomes

$$15\mathbf{I}_2 + 40\mathbf{I}_2 - \mathbf{V}_2 = 0 \quad \Rightarrow \quad \mathbf{V}_2 = 55\mathbf{I}_2 \quad (13.9.6)$$

Substituting Eq. (13.9.6) in Eq. (13.9.5),

$$-165\mathbf{I}_2 = 120 \quad \Rightarrow \quad \mathbf{I}_2 = -\frac{120}{165} = -0.7272 \text{ A}$$

The power absorbed by the 10-Ω resistor is

$$P = (-0.7272)^2(10) = 5.3 \text{ W}$$

Practice Problem 13.9

Find $\mathbf{V}_o$ in the circuit of Fig. 13.40.

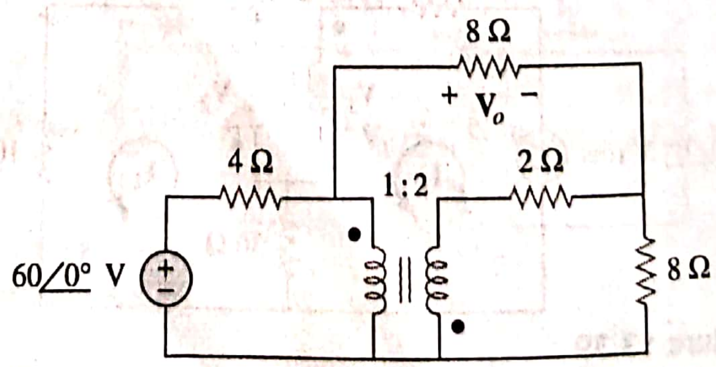

Figure 13.40
For Practice Prob. 13.9.

Answer: 24 V.

Example 13.4

In the circuit of Fig. 13.24, calculate the input impedance and current $\mathbf{I}_1$. Take $\mathbf{Z}_1 = 60 - j100\ \Omega$, $\mathbf{Z}_2 = 30 + j40\ \Omega$, and $\mathbf{Z}_L = 80 + j60\ \Omega$.

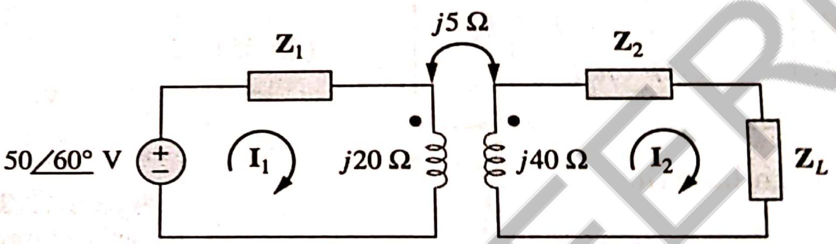

Figure 13.24
For Example 13.4.

Solution:
From Eq. (13.41),

$$\mathbf{Z}_{in} = \mathbf{Z}_1 + j20 + \frac{(5)^2}{j40 + \mathbf{Z}_2 + \mathbf{Z}_L}$$

$$= 60 - j100 + j20 + \frac{25}{110 + j140}$$

$$= 60 - j80 + 0.14\angle{-51.84^\circ}$$

$$= 60.09 - j80.11 = 100.14\angle{-53.1^\circ}\ \Omega$$

Thus,

$$\mathbf{I}_1 = \frac{\mathbf{V}}{\mathbf{Z}_{in}} = \frac{50\angle 60^\circ}{100.14\angle -53.1^\circ} = 0.5\angle 113.1^\circ \text{ A}$$

Practice Problem 13.4

Find the input impedance of the circuit in Fig. 13.25 and the current from the voltage source.

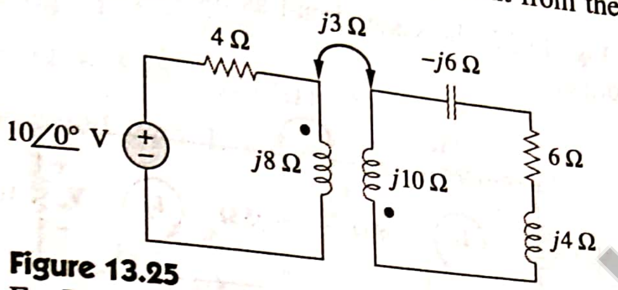

Figure 13.25
For Practice Prob. 13.4.

Answer: $8.58\angle 58.05^\circ\ \Omega$, $1.165\angle -58.05^\circ$ A.

Example 13.5

Determine the T-equivalent circuit of the linear transformer in Fig. 13.26(a).

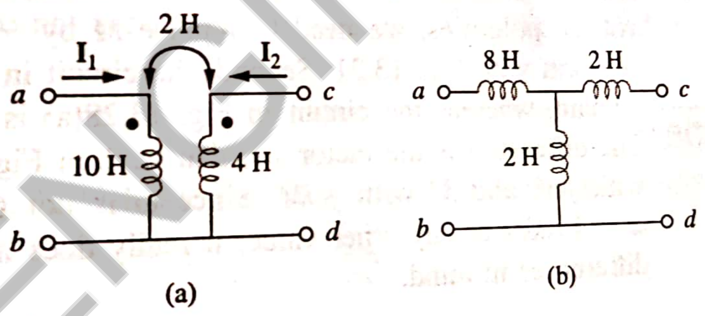

Figure 13.26
For Example 13.5: (a) a linear transformer, (b) its T-equivalent circuit.

Solution:
Given that $L_1 = 10$, $L_2 = 4$, and $M = 2$, the T-equivalent network has the following parameters:

$$L_a = L_1 - M = 10 - 2 = 8 \text{ H}$$

$$L_b = L_2 - M = 4 - 2 = 2 \text{ H}, \qquad L_c = M = 2 \text{ H}$$

The T-equivalent circuit is shown in Fig. 13.26(b). We have assumed that reference directions for currents and voltage polarities in the primary and secondary windings conform to those in Fig. 13.21. Otherwise, we may need to replace M with $-M$, as Example 13.6 illustrates.

Practice Problem 13.5

For the linear transformer in Fig. 13.26(a), find the Π equivalent network.

Answer: $L_A = 18$ H, $L_B = 4.5$ H, $L_C = 18$ H.

Example 13.6

Solve for $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{V}_o$ in Fig. 13.27 (the same circuit as for Practice Prob. 13.1) using the T-equivalent circuit for the linear transformer.

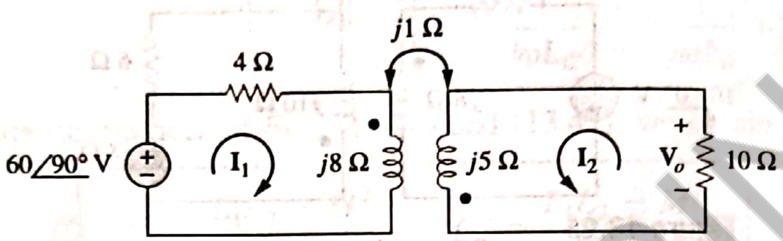

Figure 13.27
For Example 13.6.

Solution:
Notice that the circuit in Fig. 13.27 is the same as that in Fig. 13.10 except that the reference direction for current $\mathbf{I}_2$ has been reversed, just to make the reference directions for the currents for the magnetically coupled coils conform with those in Fig. 13.21.

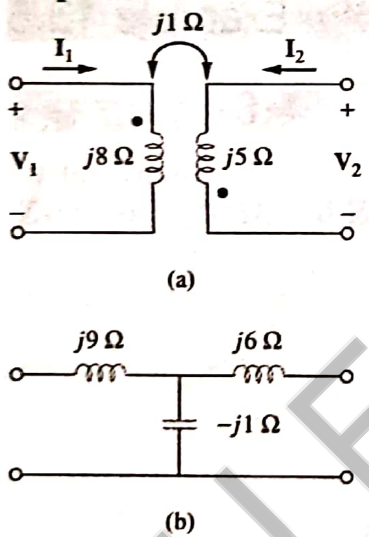

Figure 13.28
For Example 13.6: (a) circuit for coupled coils of Fig. 13.27, (b) T-equivalent circuit.

We need to replace the magnetically coupled coils with the T-equivalent circuit. The relevant portion of the circuit in Fig. 13.27 is shown in Fig. 13.28(a). Comparing Fig. 13.28(a) with Fig. 13.21 shows that there are two differences. First, due to the current reference directions and voltage polarities, we need to replace M by $-M$ to make Fig. 13.28(a) conform with Fig. 13.21. Second, the circuit in Fig. 13.21 is in the time-domain, whereas the circuit in Fig. 13.28(a) is in the frequency-domain. The difference is the factor $j\omega$; that is, L in Fig. 13.21 has been replaced with $j\omega L$ and M with $j\omega M$. Since ω is not specified, we can assume $\omega = 1$ rad/s or any other value; it really does not matter. With these two differences in mind,

$$L_a = L_1 - (-M) = 8 + 1 = 9\text{ H}$$

$$L_b = L_2 - (-M) = 5 + 1 = 6\text{ H}, \qquad L_c = -M = -1\text{ H}$$

Thus, the T-equivalent circuit for the coupled coils is as shown in Fig. 13.28(b).

Inserting the T-equivalent circuit in Fig. 13.28(b) to replace the two coils in Fig. 13.27 gives the equivalent circuit in Fig. 13.29, which can be solved using nodal or mesh analysis. Applying mesh analysis, we obtain

$$j6 = \mathbf{I}_1(4 + j9 - j1) + \mathbf{I}_2(-j1) \tag{13.6.1}$$

and

$$0 = \mathbf{I}_1(-j1) + \mathbf{I}_2(10 + j6 - j1) \tag{13.6.2}$$

From Eq. (13.6.2),

$$\mathbf{I}_1 = \frac{(10 + j5)}{j}\mathbf{I}_2 = (5 - j10)\mathbf{I}_2 \tag{13.6.3}$$

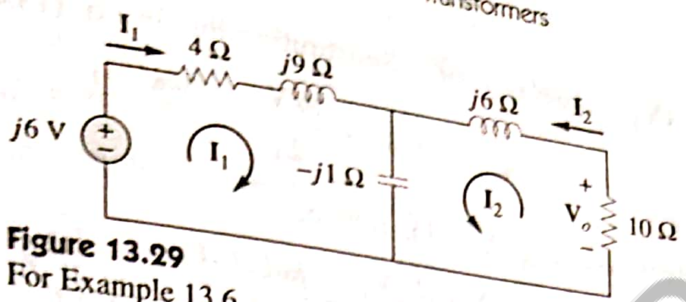

Figure 13.29
For Example 13.6.

Substituting Eq. (13.6.3) into Eq. (13.6.1) gives

$$j6 = (4 + j8)(5 - j10)\mathbf{I}_2 - j\mathbf{I}_2 = (100 - j)\mathbf{I}_2 \simeq 100\mathbf{I}_2$$

Since 100 is very large compared with 1, the imaginary part of $(100 - j)$ can be ignored so that $100 - j \simeq 100$. Hence,

$$\mathbf{I}_2 = \frac{j6}{100} = j0.06 = 0.06\underline{/90^\circ} \text{ A}$$

From Eq. (13.6.3),

$$\mathbf{I}_1 = (5 - j10)j0.06 = 0.6 + j0.3 \text{ A}$$

and

$$\mathbf{V}_o = -10\mathbf{I}_2 = -j0.6 = 0.6\underline{/-90^\circ} \text{ V}$$

This agrees with the answer to Practice Prob. 13.1. Of course, the direction of $\mathbf{I}_2$ in Fig. 13.10 is opposite to that in Fig. 13.27. This will not affect $\mathbf{V}_o$, but the value of $\mathbf{I}_2$ in this example is the negative of that of $\mathbf{I}_2$ in Practice Prob. 13.1. The advantage of using the T-equivalent model for the magnetically coupled coils is that in Fig. 13.29 we do not need to bother with the dot on the coupled coils.

Practice Problem 13.6

Solve the problem in Example 13.1 (see Fig. 13.9) using the T-equivalent model for the magnetically coupled coils.

Answer: $13\underline{/-49.4^\circ}$ A, $2.91\underline{/14.04^\circ}$ A.