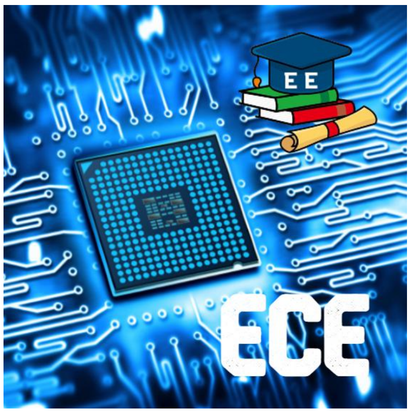


EE
ECE

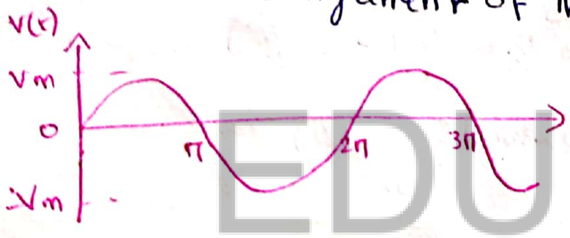
Sinusoid → is a signal that has the form of the cosine or sine function.

$$\boxed{v(t) = V_m \sin\omega t}$$

V_m = amplitude of the sinusoid.

ω = angular frequency in radian/s

ωt = the argument of the sinusoid.



$$\boxed{\omega T = 2\pi}$$

$$\boxed{T = \frac{2\pi}{\omega}}$$

T = Period of sinusoid.

$$\boxed{f = 1/T} \qquad \boxed{\omega = 2\pi f}$$

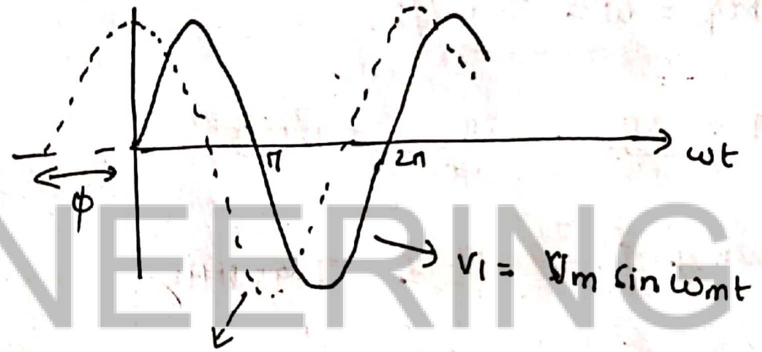
f = frequency in hertz ⇒ or 'f' is also called cycle frequency.

ω = angular frequency.

More general expression for the sinusoid.

$$\boxed{v(t) = V_m \sin(\omega t + \phi)}$$

ϕ = Phase.



The starting point of v_2 occurs first in time, therefore we say that v_2 leads v_1 by ϕ (or) v_1 lags v_2 by ϕ

If $\phi = 0$ ⇒ v_1 & v_2 are in-phase.

$\phi \neq 0$ ⇒ v_1 & v_2 are out-of phase.

1) Find the amplitude, phase & frequency of the sinusoid.

$$v(t) = 12\cos(50t + 10^\circ)$$

Amplitude = V_m = 12 V

Phase = ϕ = 10°

Angular freq = ω = 50 rad/s

Period $T = \frac{2\pi}{\omega} = \frac{2\pi}{50} = 0.1257s$

Frequency 'f' = $1/T$ = 7.958 Hz.

2) Given the sinusoid $5\sin(4\pi t - 60^\circ)$, calculate its amplitude, phase, angular frequency, period and frequency.

Amplitude = 5V

Phase = -60°

Angular frequency $\omega = 4\pi = 4 \times 3.14 = 12.56$ rad/sec

frequency $f = \omega/2\pi = \frac{12.56}{2\times\pi} = \underline{\underline{2Hz}}$

1) Calculate the phase angle between $V_1 = -10\cos(\omega t + 50^\circ)$ and $V_2 = 12\sin(\omega t - 10^\circ)$. State which sinusoid is leading.

$$V_1 = -10\cos(\omega t + 50^\circ)$$

$$= 10\cos(\omega t + 50^\circ - 180^\circ)$$

$$= 10\cos(\omega t - 130^\circ) \quad \text{--- (1)}$$

(or) ~~$V_1 = 10\cos(\omega t + 230^\circ)$~~

$V_2 = 12\sin(\omega t - 10^\circ) = 12\cos(\omega t - 10^\circ - 90^\circ)$

$V_2 = 12\cos(\omega t - 100^\circ)$ --- (2). ~~or~~

From (1) & (2) Phase difference between V_1 & V_2 is 30°.

V_2 leads V_1 by 30°.

Phasors:

A Phasor is a Complex number that represents the amplitude and phase of a sinusoid.

Rectangular form.

$\boxed{Z = x + jy}$ ⇐ Complex no in rectangular form.

$j = \sqrt{-1}$

$x \Rightarrow$ real part

$y \Rightarrow$ imaginary part

Complex no. can also be written in polar form:-

Polar form:

$$\boxed{z = r\angle\phi = re^{j\phi}}$$

Complex no. 'z' can also be written in polar or exponential form.

r = magnitude of 'z'

ϕ = phase of z

$$\boxed{\begin{array}{ll} Z = x + jy & \Rightarrow \text{Rectangular form} \\ z = r\angle\phi & \Rightarrow \text{Polar form} \\ z = re^{j\phi} & \Rightarrow \text{Exponential form.} \end{array}}$$

Addition & Subtraction are Better performed in Rectangular form.

Multiplication & Division are better performed in Polar form

Addition:

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$$

Subtraction:

$$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$$

Multiplication:

$$z_1 z_2 = r_1 r_2 \angle \phi_1 + \phi_2$$

Division:

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \angle \phi_1 - \phi_2$$

Reciprocal:

$$\frac{1}{z} = \frac{1}{r}\angle{-\phi}$$

Squareroot

$$\sqrt{z} = \sqrt{r}\,\angle{\phi/2}$$

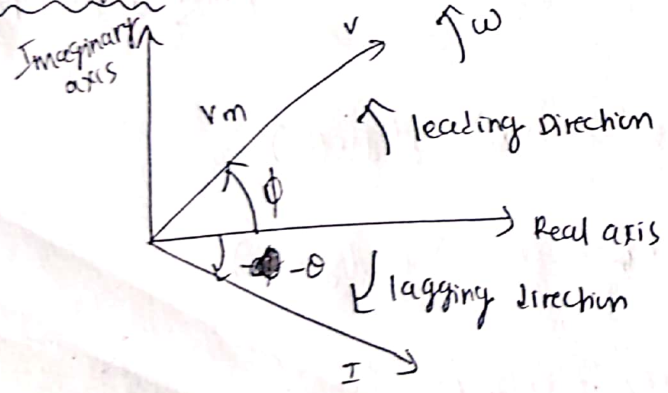
Complex Conjugate:

$$z^* = x - jy = r\angle{-\phi} = re^{-j\phi}$$

$v(t) = V_m \cos(\omega t + \phi)$ = Time Domain representation

$V = V_m \angle\phi$ = Phasor domain representation

Phasor-Diagram:



Phasor Diagram showing

$V = V_m \angle\phi$ & $I = I_m \angle{-\theta}$

Problems:

Transform the Sinusoids to phasors.

① $i = 6\cos(50t - 40^\circ)\,A$

$$\boxed{I = 6\angle{-40^\circ}\,A}$$

② $v = -4\sin(30t + 50^\circ)\,V$

$= 4\cos(30t + 50^\circ + 90^\circ)$ (Since $-\sin A = \cos(A + 90^\circ)$)

$= 4\cos(30t + 140^\circ)\,V$

$$\boxed{V = 4\angle{140^\circ}\,V}$$

③ $v = -7\cos(2t + 40^\circ)\,V$

$= 7\cos(2t + 40 + 180)$

$= 7\cos(2t + 220)\,V$

$$\boxed{V = 7\angle{220^\circ}\,V}$$

(4) $i = 4\sin(10t + 10^\circ)\,A$

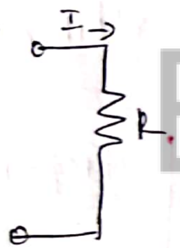
$= 4\cos(10t + 10^\circ - 90^\circ)\,A$

$= 4\cos(10t - 80^\circ)\,A$

$$\boxed{i = 4\angle{-80^\circ}\,A}$$

Phasor relationships for Circuit Elements:-

Resistor



$i = I_m\cos(\omega t + \phi)$ = current

$V = iR$

$V = R I_m \cos(\omega t + \phi)$

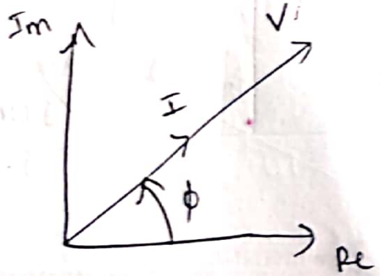
Phasor form:-

$V = R I_m \angle\phi$

$I = I_m\angle\phi$

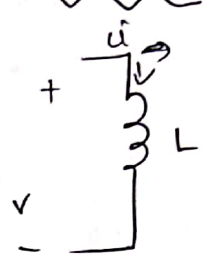
$V = IR.$

Phasor Diagram for Resistor



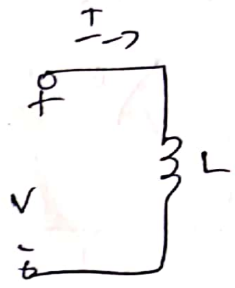
$\boxed{\text{Voltage and current are in-phase.}}$

For Inductor:-



Time Domain

$$\boxed{V = L\,\frac{di}{dt}}$$



freq-domain

$$\boxed{V = j\omega L I}$$

Assume current through Inductor is $i = I_m\cos(\omega t + \phi)$

$V = L\,\frac{di}{dt} = -\omega L I_m \sin(\omega t + \phi)$

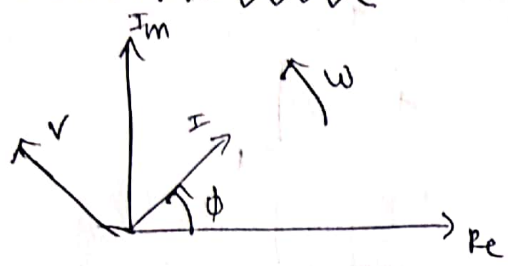
~~$\therefore -\sin A =$~~ $\cos(A + 90^\circ)$

$V = \omega L I_m \cos(\omega t + \phi + 90^\circ)$ $\quad [\because -\sin A = \cos(A + 90^\circ)]$

$$\boxed{V = \omega L I_m \angle{\phi + 90^\circ}}$$

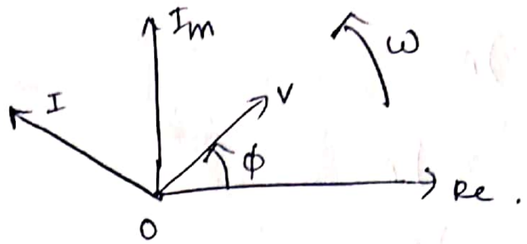
$$\boxed{I_m\angle\phi = I}$$

Phasor Diagram for Inductor:-



Voltage & current are 90° out of phase.

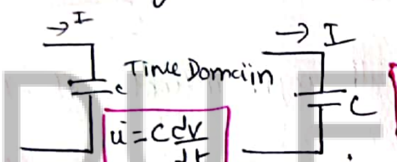
Current lags voltage by 90°.



Current & voltage are 90° out of phase.

Current leads the voltage by 90°.

For Capacitor:-



$\boxed{i = C\frac{dv}{dt}}$ $\boxed{I = j\omega CV}$

assume the voltage across it is $v = V_m \cos(\omega t + \phi)$

Current through the capacitor is

$$i = C\frac{dv}{dt}$$

$$\boxed{I = j\omega CV \Rightarrow V = \frac{I}{j\omega C}}$$

Summary of voltage current relationships.

Element	Time Domain	Frequency Domain
R	$v = Ri$	$V = RI$
L	$v = L\,di/dt$	$V = j\omega LI$
C	$i = C\,dv/dt$	$V = I/j\omega C$

Problem:-

The voltage $v = 12\cos(60t + 45^\circ)$ is applied to a 0.1H Inductor, Find the Steady State current through the inductor.

Soln

$V = j\omega L I \qquad \omega = 60$ rad/s

$V = 12\angle 45^\circ$ V

$$I = \frac{V}{j\omega L} = \frac{12\angle 45^\circ}{j60 \times 0.1} = \frac{12\angle 45^\circ}{6\angle 90^\circ}$$

$$\boxed{I = 2\angle -45^\circ \text{ A}}$$

$$\boxed{I = 2\cos(60t - 45^\circ)\text{A}}$$

(Time Domain)

If voltage $v = 6\cos(100t - 30^\circ)$ is applied to a $50\mu F$ capacitor, calculate the current through it.

$V = I/j\omega C$

$I = V\, j\omega C$

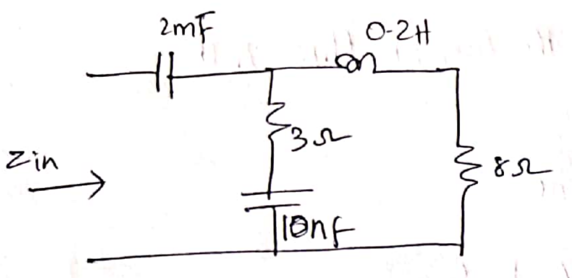
$= 6\angle -30^\circ \times j\,100 \times 50 \times 10^{-6}$

$$\boxed{I = 30\cos(100t + 60^\circ)\text{mA}}$$

Impedance & Admittance of Passive Elements:-

Element	Impedance	Admittance
R	$Z = R$	$Y = 1/R$
L	$Z = j\omega L$	$Y = 1/j\omega L$
C	$Z = 1/j\omega C$	$Y = j\omega C$.

Find the input impedance of the circuit; operates at $\omega = 50$ rad/s



$$Z_1 = \frac{1}{j\omega C} = \frac{1}{j50 \times 2 \times 10^{-3}} = -j10\,\Omega$$

$$Z_2 = 3 + \frac{1}{j\omega C} = 3 + \frac{1}{j50 \times 10 \times 10^{-3}} = (3-j2)\,\Omega$$

$$Z_3 = 8 + j\omega L = 8 + j50 \times 0.2 = (8+j10)\,\Omega$$

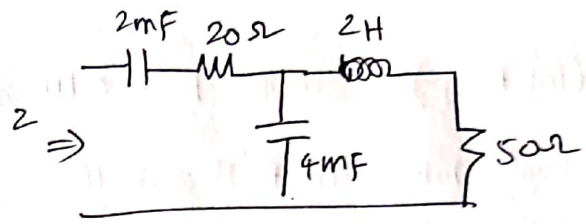
Input Impedance

$$Z = Z_1 + Z_2 \parallel Z_3$$

$$= -j10 + \frac{(3-j2)(8+j10)}{11+j8}$$

$$\boxed{Z_{in} = 3.22 - j11.07\,\Omega}$$

Determine the input impedance of the circuit at $\omega = 10$ rad/s



$Z = 32.38 - j73.76\,\Omega$

$$Z_1 = 20 + \frac{1}{j\omega C} = 20 + \frac{1}{j10 \times 2 \times 10^{-3}} = 20 - 50j$$

$$Z_2 = \frac{1}{j\omega C} = \frac{1}{j10 \times 4 \times 10^{-3}} = -25j$$

$$Z_3 = 50 + j\omega L = 50 + j \times 10 \times 2 = 50 + 20j$$

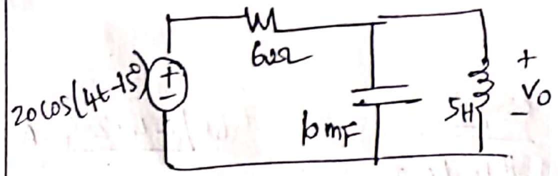
$$Z_1 + (Z_2 \parallel Z_3)$$

$$(20-50j) + \frac{(-25j)(50+20j)}{(-25j+50+20j)} = \frac{1346.29\angle -68.19}{50.24\angle -5.71}$$

$$= 53.85\angle -68.1 + 26.79\angle -63.02$$

$$\boxed{Z = 32.23 - j73.83\,\Omega}$$

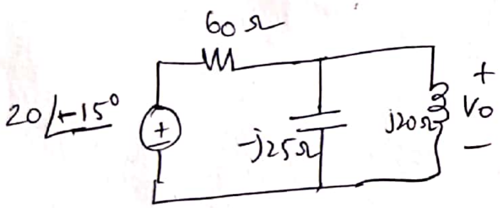
Determine $v_o(t)$ in the circuit.



$V_s = 20\cos(4t-15^\circ) = 20\angle{-15^\circ}\,V$, $\omega = 4$

$10mF = \frac{1}{j\omega C} = \frac{1}{j4 \times 10 \times 10^{-3}} = -j25\,\Omega$

$5H = j\omega L = j4 \times 5 = j20\,\Omega$



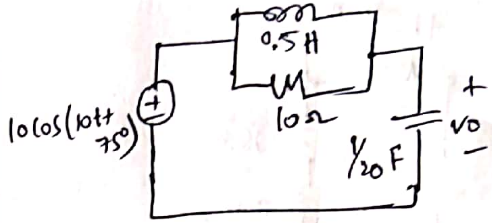
$Z_1 = 60\,\Omega$ $\quad Z_2 = -j25 \,||\, j25 = j100\,\Omega$

$V_0 = \frac{Z_2}{Z_1+Z_2} V_s = \frac{j100}{60+100j}(20\angle{-15^\circ})$

$= (0.8575\angle{30.96^\circ})(20\angle{-15^\circ}) = 17.15\angle{15.96^\circ}\,V$

$\boxed{\text{Time Domain} \Rightarrow V_o(t) = 17.15\cos(4t+15.96^\circ)\,V}$

(2) Calculate 'V_o'



$V_s = 10\cos(10t-15^\circ)\,V$ $\qquad (0.5H \,||\, 10\Omega) + (1/20\,F)$

$V_s = 10\angle{-15^\circ}\,V$

$\omega = 10$ rad/sec

~~$Z_1 = 10\Omega$~~ , ~~$Z_2 =$~~

$Z_1 = (0.5H \,||\, 10\Omega)$ $\qquad Z_2 = (1/20\,F)$

$0.5H \Rightarrow j\omega L = j10 \times 0.5 =$

$Z_1 = j\omega L = j10 \times 0.5 = 5j$

$Z_2 = 10\Omega$

$Z_3 = 1/j\omega C = 1/j10 \times 1/20 = 1/0.5j = -2j$

$Z = (Z_1 || Z_2) + Z_3$

$(5j || 10) + (-2j)$

$= \frac{(10 \times j5)}{(10+j5)} - 2j$

$= \left(\frac{j50}{10+j5}\right) - 2j$

$2 + j4 - 2j$

$\boxed{Z_{in} = 2+j2\,\Omega}$

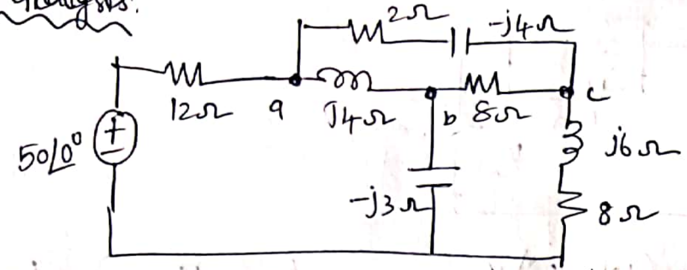
$I = V/Z_{in} = \frac{10\angle{75^\circ}}{2+j2} = 3.06+1.76j$

$V_o = I \times Z_3 = (3.06+1.76j)(-2j)$

$V_o(t) = 7.06\angle{-60^\circ}$ (or) ~~7.06cos(10t~~

$V_o(t) = 7.06\cos(10t - 60^\circ)\,V$

Find the current 'I' in the circuit
Mesh Analysis:



Delta network connected to nodes a, b, & c can be converted to the Y network

$$Z_{an} = \frac{j4(2-j4)}{j4+2-j4+8} = \frac{4(4+j2)}{10} = (1.6+j0.8)\,\Omega$$

$$Z_{bn} = \frac{j4(8)}{10} = j3.2\,\Omega,$$

$$Z_{cn} = \frac{8(2-j4)}{10} = (1.6-j3.2)\,\Omega.$$

The total impedance at the source terminals is

$$Z = 12 + Z_{an} + (Z_{bn} - j3)\,||\,(Z_{cn} + j6 + 8)$$

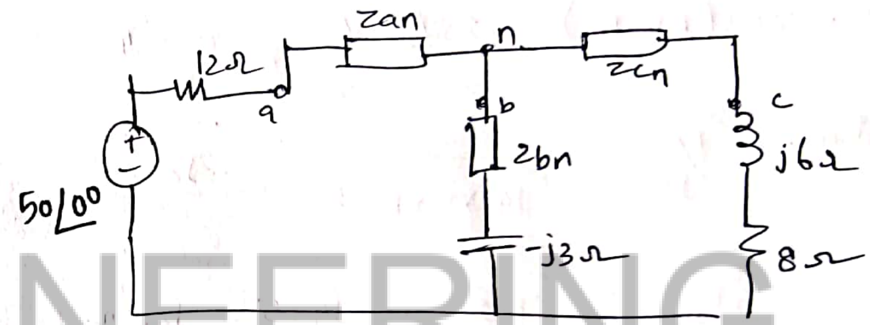
$$= 12 + 1.6 + j0.8 + (j0.2)\,||\,(9.6 + j2.8)$$

$$= 13.6 + j0.8 + \frac{j0.2(9.6+j2.8)}{9.6+j3}$$

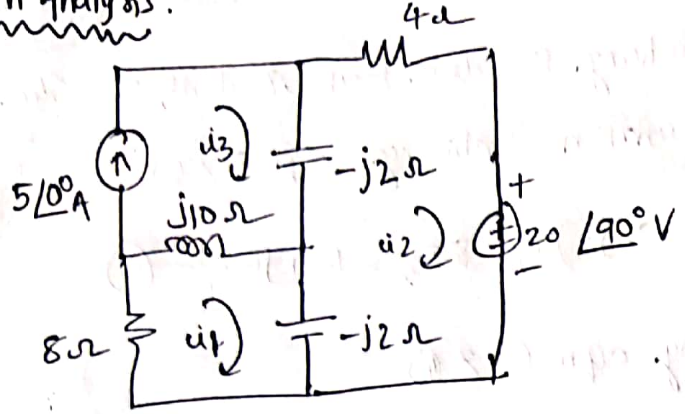
$$= 13.6 + j1 = 13.64\,\angle 4.204^\circ\,\Omega$$

The desired current is

$$I = V/Z = \frac{50\angle 0^\circ}{13.64\angle 4.204^\circ} = 3.666\,\angle -4.204^\circ\,A$$



Mesh Analysis:



KVL @ loop 1

$$(8+j10-j2)I_1-(-j2)I_2-j10I_3=0 \quad \text{--- (1)}$$

KVL @ loop 2

$$(4-j2-j2)I_2-(-j2)I_1-(-j2)I_3+20\angle 90^\circ=0 \quad \text{--- (2)}$$

From loop 3

$I_3 = 5\angle 0^\circ$ -(3) Sub this in (1) & (2)

$$(8+j8)I_1+j2I_2=j50 \quad \text{--- (4)}$$

$$j2I_1+(4-j4)I_2=-j20-j10 \quad \text{--- (5)}$$

$$\begin{bmatrix} 8+j8 & j2 \\ j2 & 4-j4 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}=\begin{bmatrix} j50 \\ -j30 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 8+j8 & j2 \\ j2 & 4-j4 \end{vmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}=\begin{bmatrix} j50 \\ -j30 \end{bmatrix}$$

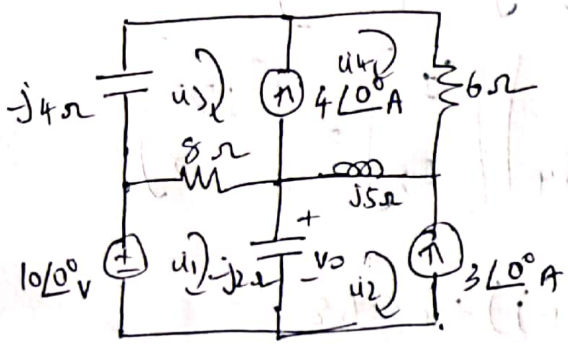
$$= 32(1+j)(1-j)+4 = 68$$

$$\Delta_2 = \begin{vmatrix} 8+j8 & j50 \\ j2 & -j30 \end{vmatrix} = 340-j240 = 416.17\angle -35.22^\circ$$

$$I_2 = \Delta_2/\Delta = \frac{416.17\angle -35.22^\circ}{68} = 6.12\angle -35.22^\circ \text{A}$$

$$\boxed{I_0 = -I_2 = 6.12\angle 144.78^\circ \text{A}}$$

② Solve for Vo in the fig.



for loop1

$$-10+(8-j2)I_1-(-j2)I_2-8I_3=0$$

$$(8-j2)I_1+j2I_2-8I_3=10 \quad --①$$

for loop2

$$i_2=-3 \quad --②$$

for Super mesh

$$(8-j4)I_3-8I_1+(6+j5)I_4-j5I_2=0 \quad --③$$

From loop 3 & 4

$$I_4=I_3+4 \quad --④$$

Instead of solving the above four equations, we reduce them to two by elimination combining eqn ① & ②

$$(8-j2)I_1-8I_3=10+j6 \quad --⑤$$

by combining eqn ③ & ④

$$-8I_1+(14+j)I_3=-24-j35 \quad --⑥$$

$$\begin{bmatrix} 8-j2 & -8 \\ -8 & 14+j \end{bmatrix}\begin{bmatrix} I_1 \\ I_3 \end{bmatrix}=\begin{bmatrix} 10+j6 \\ -24-j35 \end{bmatrix}$$

$$\Delta=\begin{vmatrix} 8-j2 & -8 \\ -8 & 14+j \end{vmatrix}=112+j8-j28+2-64=50-j20$$

$$\Delta_1=\begin{vmatrix} 10+j6 & -8 \\ -24-j35 & 14+j \end{vmatrix}=140+j10+j84-6-192-j280$$

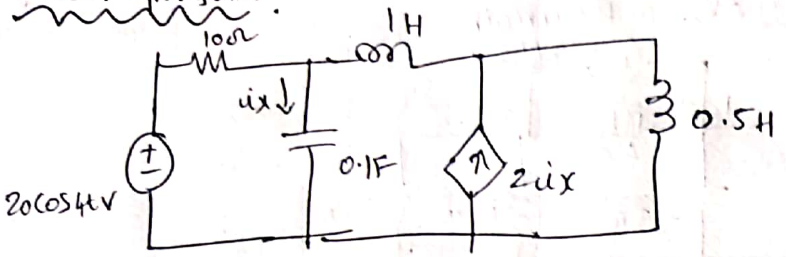
$$=-58-j186$$

$$I_1=\Delta_1/\Delta=\frac{-58-j186}{50-j20}=3.618\angle 274.5^\circ A$$

$$\text{Voltage } V_o=-j2(I_1-I_2)=-j2(3.618\angle 274.5^\circ+3)$$

$$=-7.2134-j6.568=9.756\angle 222.32^\circ V$$

Nodal Analysis:



First convert to freq. Domain.

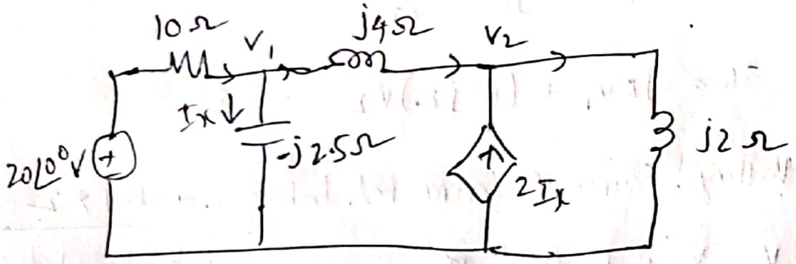
$20\cos 4t = 20\angle 0^\circ$, $\omega = 4$ rad/s.

$1H \Rightarrow j\omega L = j4$

$0.5H \Rightarrow j\omega L = j2$

$0.1F = 1/j\omega C = -j2.5$

Freq. Domain Equivalent circuit.



Applying KCL at node 1

$$\frac{20-V_1}{10} = \frac{V_1}{-j2.5} + \frac{V_1-V_2}{j4}$$

$$(1+j1.5)V_1 + j2.5V_2 = 20 \quad \text{--- (1)}$$

At node 2

$$2I_x + \frac{V_1-V_2}{j4} = V_2/j2$$

But $I_x = V_1/-j2.5$

$$\frac{2V_1}{-j2.5} + \frac{V_1-V_2}{j4} = V_2/j2$$

Simplifying we get

$$11V_1 + 15V_2 = 0 \quad \text{--- (2)}.$$

$$\begin{bmatrix} 1+j1.5 & j2.5 \\ 11 & 15 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 1+j1.5 & j2.5 \\ 11 & 15 \end{vmatrix} = 15-j5$$

$$\Delta_1 = \begin{vmatrix} 20 & j2.5 \\ 0 & 15 \end{vmatrix} = 300, \quad \Delta_2 = \begin{vmatrix} 1+j1.52 & 0 \\ 11 & 0 \end{vmatrix} = -220$$

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{300}{15-j5} = 18.97\angle 18.43^\circ\ V$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{-220}{15-j5} = 13.91\angle 198.3^\circ\ V$$

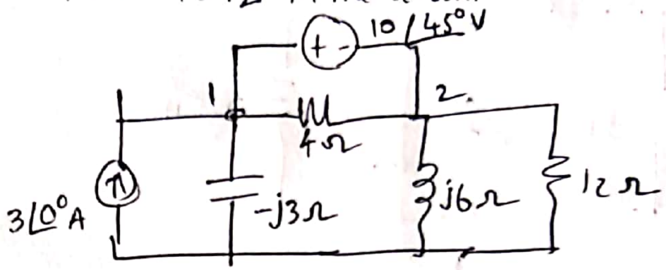
The current I_x is given by

$$I_x = \frac{V_1}{-j2.5} = \frac{18.97\angle 18.43^\circ}{2.5\angle -90^\circ} =$$

$$= 7.59\angle 108.4^\circ\ A$$

$$\boxed{i_x = 7.59\cos(4t + 108.4^\circ)A}$$

(2) Compute V_1 & V_2 in the circuit.



Node 1 & 2 form a supernode.

Applying KCL

$$3 = \frac{V_1}{-j3} + \frac{V_2}{j6} + \frac{V_2}{12}$$

$$36 = j4V_1 + (1-j2)V_2 \quad \text{--- (1)}$$

But a voltage source is connected between node 1 & 2

So $V_1 = V_2 + 10\angle 45^\circ$ --- (2) $\boxed{V_1 - V_2 = 10\angle 45^\circ}$

Subs (2) in (1)

$$36 - 40\angle 135^\circ = (1+j2)V_2 \quad \Big| \quad V_2 = 31.41\angle -87.18^\circ V$$

From (2)

$$V_1 = V_2 + 10\angle 45^\circ = 25.78\angle -70.48^\circ V \quad \boxed{V_1 = 25.78\angle -70.48^\circ V}$$

Instantaneous power & Average power:

Instantaneous power = instantaneous voltage × instantaneous current

$$\boxed{P(t) = v(t)\,i(t)}$$

The instantaneous power (watts) is the power at any instant of time.

Let

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_i)$$

V_m & I_m ⇒ are the amplitudes

θ_v & θ_i ⇒ Phase angle of voltage & current respectively.

$$P(t) = v(t)\,i(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$

$$\cos A \cos B = \frac{1}{2}\{\cos(A-B) + \cos[A+B]\}$$

$$P(t) = \underbrace{\frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)}_{\text{Constant or time dependent}} + \underbrace{\frac{1}{2} V_m I_m (2\omega t + \theta_v + \theta_i)}_{\text{Sinusoidal fn}} \quad \text{--- (1)}$$

$P(t)$ can be positive or negative

If $p(t)$ = positive ⇒ Power is absorbed by the circuit

if $p(t)$ = negative ⇒ Power is absorbed by the source.

Average power:

average power in watts, is the average of the instantaneous power over one period.

$$\boxed{P = \frac{1}{T}\int_0^T p(t)\,dt} \quad \text{--- (2)}$$

Subs ② in ①

$$P = \frac{1}{T}\int_0^T \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)\,dt + \frac{1}{T}\int_0^T \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)\,dt$$

$$= \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)\,\frac{1}{T}\int_0^T dt + \frac{1}{2} V_m I_m \frac{1}{T}\int_0^T \cos(2\omega t + \theta_v + \theta_i)\,dt$$

Constant

Sinusoid.

First integral is constant, second is sinusoid, average of a sinusoid over a its period is zero.

So Average power is

$$\boxed{P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)}$$

Since $\cos(\theta_v - \theta_i) = \cos(\theta_i - \theta_v) \Rightarrow$ difference in the phases of the voltage & current.

$P(t)$ = time varying

P = does not depend on time

for instantaneous power we must know $v(t)$ & $i(t)$ (time varying)

But average power can be calculated in when the voltage & current are expressed in ~~freq~~ time domain. or freq domain

eg. $V(t) = V_m \angle \theta_v$

$I(t) = I_m \angle \theta_i$

$$P = \frac{1}{2} VI = \frac{1}{2} V_m I_m \angle \theta_v - \theta_i$$

$$= \frac{1}{2} V_m I_m [\cos(\theta_v - \theta_i) + j\sin(\theta_v - \theta_i)]$$

Real part of this equation in average power.

$P = \frac{1}{2} Re[V I^*]$

$$\boxed{P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)}$$

When $\theta_v = \theta_i$

$P = \frac{1}{2} V_m I_m = \frac{1}{2} I_m^2 R = \frac{1}{2} |I|^2 R$

(Shows the circuit is Purely resistive)

When $\theta_v - \theta_i = 90^\circ$

$P = \frac{1}{2} V_m I_m \cos 90^\circ = 0$

Shows that the circuit is purely reactive. Circuit absorbs no average power

A resistive load (R) absorbs power at all times, while a reactive load (L or C) absorbs zero average power

① Problem

Given that $v(t) = 120 \cos(377t + 45^\circ) V$

$i(t) = 10 \cos(377t - 10^\circ) A$

Find the instantaneous power & average power absorbed by the passive linear ntk.

Instantaneous power

$P = vi = 1200 \cos(377t + 45^\circ) \cos(377t - 10^\circ)$

$p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$

$\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$= 600[\cos(754t + 35^\circ) + \cos 55^\circ]$

$$\boxed{p(t) = 344.2 + 600 \cos(754t + 135^\circ) W}$$

average Power P_{avg}

$P_{avg} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2}(120) 10 \cos(45^\circ - (-10^\circ))$

$= 600 \cos 55^\circ = \underline{\underline{344.2 W}}$

(2) Calculate the instantaneous power & average power absorbed by the passive linear ntk.

$$v(t) = 80\cos(10t + 20^\circ)V$$

$$i(t) = 15\sin(10t + 60^\circ)V$$

i(t) can be written as

$$i(t) = 15\cos(10t + 60^\circ - 90^\circ)V$$

$$i(t) = 15\cos(10t - 30^\circ)V$$

$$p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

$$= \frac{1}{2} 80 \times 15 \cos(20 - (-30)) + \frac{1}{2} 80 \times 15 \cos(2 \times 10t + 20 - 30)$$

$$= \cancel{\tfrac{1}{2}}\, 600 \cos(50^\circ) + \cancel{\tfrac{1}{2}}\, 600 \cos(20t - 10^\circ)$$

$$\boxed{p(t) = 385.7 + 600(20t - 10^\circ)\ W}$$

Average Power Pavg

$$P_{avg} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$= \frac{1}{2} \times 80 \times 15 \cos(20 - (-30))$$

$$= 600(\cos(50))$$

$$\boxed{P_{avg} = 385.7\ W}$$

(3) Calculate the average power absorbed by an impedance $Z = 30 - j70\ \Omega$ when a voltage $V = 120\angle 0^\circ$ is applied across it.

$$I = V/Z = \frac{120\angle 0^\circ}{30 - j70} = \frac{120\angle 0^\circ}{76.16\angle -66.80^\circ} = 1.576\angle 66.8^\circ A$$

$$P_{avg} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2}(120)(1.576)\cos(0 - 66.8^\circ)$$

$$\boxed{P_{avg} = 37.24W}$$

(4) A current $I = 10\angle 30^\circ$ flows through an impedance $Z = 20\angle -22^\circ \Omega$, Find the average power delivered to the impedance.

$$V = I \cdot Z = 10\angle 30^\circ \times 20\angle -22^\circ.$$

$$V = \cancel{\ } 200\angle 8^\circ \text{ V}$$

$$P_{avg} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$= \frac{1}{2} \times 200 \times 10 \cos(8 - 30^\circ)$$

$$= 1000 \cos(-22^\circ)$$

$$\boxed{P_{avg} = 927.2 \text{ W}}$$

(5) Find the avg. power supplied by the source & the avg power ~~deliv~~ absorbed by the resistor

$$I = \frac{5\angle 30^\circ}{4 - j2} = \frac{5\angle 30^\circ}{4.472\angle -26.57} = 1.118\angle 56.57^\circ \text{A}$$

$$P_{avg} = \frac{1}{2}(5)(1.118)\cos(30^\circ - 56.57^\circ) = 2.5\text{W}.$$

Current through resistor is

$$I_R = I = 1.118\angle 56.57^\circ \text{ A}$$

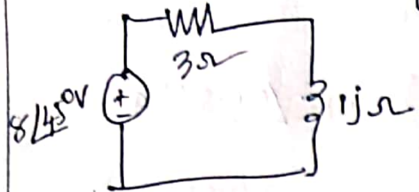
$$V_R = 4 I_R = 4.472\angle 56.57^\circ \text{V}$$

Power absorbed by resistor is

$$P = \frac{1}{2}(4.472)(1.118) = 2.5\text{W}$$

(Since phase diff bet voltage & current is zero) $(\theta_v - \theta_i = 0)$

which is the same as the average power delivered by ~~the~~ zero average power is absorbed by the capacitor.



Calculate the power (avg) absorbed by the resistor & inductor. Find the avg power supplied by voltage source.

$$I = \frac{8\angle 45^\circ V}{3+j1} = \frac{8\angle 45^\circ}{3.16\angle 18.43^\circ} = 2.53\angle 26.57^\circ A$$

Parg by Supply..

$$P_{avg} = \frac{1}{2} \times 8 \times 2.53 \cos(45^\circ - 26.57^\circ)$$

$$\boxed{P_{avg(supply)} = 9.6W}$$

Current through Resistor

$$\cancel{I = \frac{8\angle 45^\circ}{3} = 2.66\angle 45^\circ A}$$

$$\cancel{V = (2.66\angle 45^\circ) \times 3 =}$$

$$\cancel{P_{avg(res)} = \frac{1}{2} \times 8 \times 2.66 \cos(45^\circ - 45^\circ)}$$

Current through resistor $I = 2.53\angle 26.57^\circ A$

Voltage in resistor $(2.53\angle 26.57^\circ \times 3) = 7.59\angle 26.57^\circ$

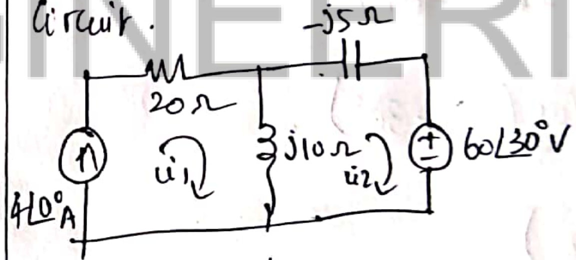
Pow in resistor $= \frac{1}{2} \times \cancel{8} \times 7.59 \times 2.53 \cos(26.57 - 26.57)$

$$= \frac{1}{2} \times 7.59 \times 2.53$$

$$\boxed{P_{avg\ res.} = 9.6W}$$

Power absorbed by Inductor is zero.

(3) Determine the average power generated by each source and the power absorbed by each passive elements of the circuit.



apply mesh/loop 1

$$i_1 = 4A$$

From mesh/loop 2

$$(j10 - j5)I_2 - j10I_1 + 60\angle 30^\circ = 0,$$

$$j5I_2 = -60\angle 30^\circ + j40 \Rightarrow I_2 = -12\angle -60^\circ + 8$$

$$= 10.58\angle 79.1^\circ A$$

for the voltage source, the current flowing from it is I_2

$$I_2 = 10.58\angle 79.1^\circ A.$$

& voltage is $60\angle 30^\circ V$ so avg power is

$$P_5 = \frac{1}{2}(60)(10.58)\cos(30^\circ - 79.1^\circ) =$$

$$= \underline{\underline{207.8W}}$$

Thus avg power is absorbed by the source, in view of the direction of I_2 & polarity of the voltage source the ckt is delivering average power to the voltage source.

for the current source: $I_1 = 4\angle 0^\circ$.

$$V_1 = 20I_1 + j10(I_1 - I_2) = 80 + j10(4 - 2 - j10.39)$$

$$= 183.9 + j20.$$

$$= 184.984\angle 6.21^\circ V$$

The avg power supplied by the current source is

$$P_1 = -\frac{1}{2}(184.984)(4)\cos(6.21^\circ - 0) = \underline{\underline{-367.8W}}$$

negative sign indicates the current source is supplying power to the circuit.

For the resistor the current through it is $I_1 = 4\angle 0^\circ$

Voltage across it is $20I_1 = 80\angle 0^\circ$;

$$P_2 = \frac{1}{2}(80)(4) = \underline{\underline{160W}}$$

For the capacitor, the current through it is $I_2 = 10.58\angle 79.1^\circ$

& the voltage across it is $-j5I_2 = (5\angle -90^\circ)(10.58\angle 79.1^\circ)$

$$= 52.9\angle 79.1^\circ - 90^\circ.$$

avg power absorbed by capacitor is

$$P_4 = \frac{1}{2}(52.9)(10.58)\cos(-90^\circ) = 0$$

Inductor, current is $(I_1 - I_2) = 2 - j10.39 = 10.58\angle -79.1^\circ$

Voltage across it is $j10(I_1-I_2) = 10.58 \angle -79.1+90^\circ$

Hence the average power absorbed by the inductor is

$$P_3 = \frac{1}{2}(105.8)(10.58)\cos 90^\circ = 0$$

Notice that the inductor & the capacitor absorb zero average power and that the tot. power supplied by the current source equals the power absorbed by the resistor & the voltage source or

$$P_1+P_2+P_3+P_4+P_5 = -367.8+160+0+0+207.8 = 0$$

indicating that power is conserved.

Effective or Rms value:

The effective value of a periodic current is the dc current that delivers the same average power to a resistor as the periodic current.

$\boxed{I_{eff} = I_{rms}}$ $\boxed{V_{eff} = V_{rms}}$

$\boxed{V_{rms} = \frac{V_m}{\sqrt{2}}}$. $\boxed{I_{rms} = \frac{I_m}{\sqrt{2}}}$

average power can be written in terms of rms values

$$\boxed{P = I^2_{rms} R = \frac{V^2_{rms}}{R}}$$

Apparent Power & Power Factor:-

Apparent power (in VA) is the product of the rms values of voltage and current.

units (Volt-Amp)

Power factor:- Ratio of ~~apparent~~ average power to apparent power

$$\boxed{P_f = \frac{P}{S} = \cos(\theta_v - \theta_i)}$$

$\theta_v - \theta_i =$ Power factor angle.

Power factor is the cosine of the phase difference between voltage & current, It is also the cosine of the angle of load

(1) A Series-Connected load draws a current $i(t) = 4\cos(100\pi t + 10^\circ)$ A when the applied voltage is $v(t) = 120\cos(100\pi t - 20^\circ)$ V, find the apparent power & the power factor of the load, Determine the element values that form the Series-connected load.

Apparent Power $S = V_{rms} \cdot I_{rms} \quad \frac{120}{\sqrt{2}} \frac{4}{\sqrt{2}} = \underline{\underline{240\text{ VA}}}$

Power factor $Pf = \cos(\theta_v - \theta_i) =$

$= \cos(-20-10) = 0.866$ (leading)

(Because current leads voltage)

$Z = V/I = \frac{120\angle -20^\circ}{4\angle 10^\circ} = 25.98 - j15\,\Omega$

$Pf = \cos(-30^\circ) = 0.866$ (leading).

load impedance 'Z' can be modeled by a 25.98 Ω resistor in Series with a capacitor

$X_c = -15 = -1/\omega C$

$C = 1/15\omega = \frac{1}{15 \times 100\pi} = 212.2\,\mu F$

(2) Obtain Power factor & the apparent power of a load whose impedance is $Z = 60 + j40\,\Omega$ $v(t) = 150\cos(377t + 10^\circ)$ V

~~Apparent power S = Vrms Irms~~ :

$I(t) = \frac{v(t)}{Z} = \frac{150\angle 10^\circ}{60 + j40} = \frac{150\angle 10^\circ}{72.11\angle 33.69}$

$I(t) = \cancel{2.08\angle -23.69}\ \ 2.08\angle -23.69$

$S = V_{rms} I_{rms} = \frac{150}{\sqrt{2}} \times \frac{2.08}{\sqrt{2}} = \underline{\underline{156\text{ VA}}}$

$Pf = \cos(\theta_v - \theta_i) =$

$= \cos(10 + 23.69) = 0.832$ (lagging)

(Because current lags voltage)

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Complex power: in (VA)

is the product of the rms voltage phasor and the complex conjugate of the rms current phasor. As a complex quantity its real part is real power 'P' & its imaginary part is reactive power Q.

$$\boxed{\text{Complex power} = S = P + jQ = \frac{1}{2} V I^* = V_{rms} I_{rms} \angle \theta_v - \theta_i}$$

$$\boxed{\text{Apparent power } S = |S| = V_{rms} I_{rms} = \sqrt{P^2 + Q^2}}$$

$$\boxed{\text{Real power} = P = Re(S) = S \cos(\theta_v - \theta_i)}$$

$$\boxed{\text{Reactive power} = Q = Im(S) = S \sin(\theta_v - \theta_i)}$$

$$\boxed{\text{Power factor} = P/S = \cos(\theta_v - \theta_i)}$$

Problem:

1. The voltage across a load is $v(t) = 60 \cos(\omega t - 10°)$ V and the current through the element in the direction of the voltage drop is $i(t) = 1.5 \cos(\omega t + 50°)$ A.

Find (a) complex & apparent powers
(b) real & reactive power
(c) power factor & the load impedance.

(a)

$V_{rms} = \frac{60}{\sqrt{2}} \angle -10°$ $\quad I_{rms} = \frac{1.5}{\sqrt{2}} \angle 50°$

Complex power $= V_{rms} I_{rms}^* \left(\frac{60}{\sqrt{2}} \angle -10°\right)\left(\frac{1.5}{\sqrt{2}} \angle -50°\right)$.

$$\boxed{\text{Complex power} = 45 \angle -60° \text{ VA}}$$

Apparent power

$$\boxed{S = |S| = 45 \text{ VA}}$$

(b) we can express the complex power in rectangular form

$$S = 45\angle -60^\circ = 45\left[\cos(-60) + j\sin(-60^\circ)\right]$$

$$= 22.5 - j38.97$$

Since $S = P + jQ$, the real power is

$$\boxed{\text{Real power } P = 22.5\,W}$$

while reactive power is

$$\boxed{Q = -38.97\,VAR}$$

(c) The power factor

$$pf = \cos(-60^\circ) = 0.5\ (\text{leading})$$

$$Z = \frac{V}{I} = \frac{60\angle -10^\circ}{1.5\angle +50^\circ} = 40\angle -60^\circ\ \Omega$$

(2) For a load $V_{rms} = 110\angle 85^\circ V$, $I_{rms} = 0.4\angle 15^\circ A$. Determine complex power, apparent power, real power, reactive power, power factor & load impedance.

Complex power

$$S = V_{rms} I_{rms}^*$$

$$S = 110\angle 85^\circ \times 0.4\angle -15^\circ$$

$$\boxed{S = 44\angle 70^\circ\ VA}$$

Apparent power:

$$\boxed{|S| = 44\ VA}$$

real power

Complex power in rectangular form

$$15.04 + j41.3\ W$$

$$S = P + jQ$$

Real power $P = 15.04W$

Reactive power $Q = 41.3\ VAR$

Power factor

$$Pf = \cos(\theta_v - \theta_i) \quad \text{or} \quad Pf = \cos(\theta)$$

from complex power

$$= \cos(85 - 15)$$

$$= \cos(70^\circ)$$

$$\boxed{Pf = 0.342 \text{ lagging.}}$$

Impedance

$$Z = \frac{V}{I} = \frac{V_{rms} \times \sqrt{2}}{I_{rms} \times \sqrt{2}} = \frac{110\angle 85}{0.4\angle 15^\circ}$$

$$= 275\angle 70^\circ\ \Omega$$

$$\boxed{Z = 94.05 + j258.4\ \Omega}$$

~~(B) For an load, Vrms = 110∠85° V, Irms = 0.4~~

(3) A load Z draws 12 KVA at a p.f of 0.856 lagging from a 120V rms sinusoidal source. Calculate the average & reactive powers delivered to the load, (b) Peak current and the (c) load Impedance.

Given that $Pf = \cos\theta = 0.856$, we obtain the power angle as $\theta = \cos^{-1} 0.856 = 31.13^\circ$

If apparent power is $S = 12{,}000\ VA$ the average or real power is

$$\boxed{P = S\cos\theta = 12000 \times 0.856 = 10.272\,KW}$$

While the Reactive power is

$$\boxed{Q = S\sin\theta = 12000 \times 0.517 = 6.204\ KVA}$$

(b) Since the PF is lagging, the complex power is

$$\boxed{S = P + jQ = 10.272 + j6.204\ KVA}$$

From $S = V_{rms} I_{rms}^*$

$$I_{rms}^* = \frac{S}{V_{rms}} = \frac{10{,}272 + j6204}{120\angle 0^\circ}$$

$$= 85.6 + j51.7\,A$$

$$= 100\angle 31.13^\circ\,A$$

Thus $I_{rms} = 100\angle -31.13^\circ$ & the peak current is

$$I_m = \sqrt{2}\, I_{rms} = \sqrt{2} \times 100 = 141.4\,A$$

The load Impedance

$$Z = \frac{V_{rms}}{I_{rms}} = \frac{120\angle 0^\circ}{100\angle -31.13^\circ} = 1.2\angle 31.13^\circ\,\Omega$$

Which is an inductive Impedance.